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REGULARIZATION OF THE SINGULAR INTEGRAL EQUATION FOR A WING IN AN UNSTEADY SUBSONIC GAS FLOW

by

Yu. A. Abramov

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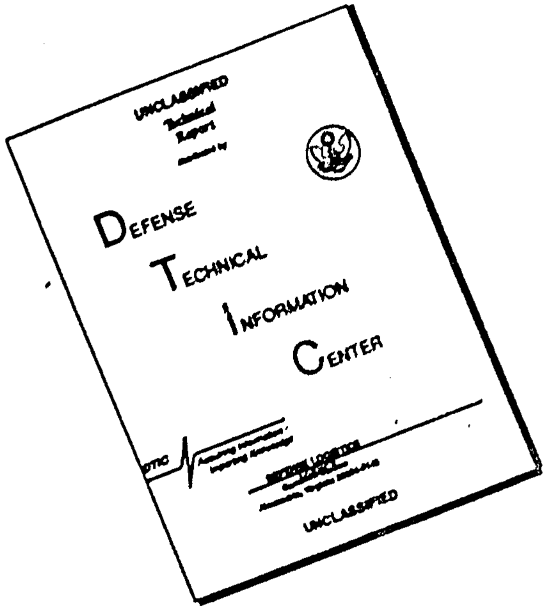

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Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
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О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α	α	*α*	Nu	Ν	ν	
Beta	Β	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	Ο	ο	
Delta	Δ	δ		Pi	Π	π	
Epsilon	Ε	ε	ϵ	Rho	Ρ	ρ	ϱ
Zeta	Ζ	ζ		Sigma	Σ	σ	ς
Eta	Η	η		Tau	Τ	τ	
Theta	Θ	θ	ϑ	Upsilon	Τ	υ	
Iota	Ι	ι		Phi	Φ	φ	ϕ
Kappa	Κ	ϰ κ	×	Chi	Χ	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	Μ	μ		Omega	Ω	ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

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REGULARIZATION OF THE SINGULAR INTEGRAL EQUATION FOR A WING IN AN UNSTEADY SUBSONIC GAS FLOW

Yu. A. Abramov

This study attempts to regularize the singular integral equation for a wing in an unsteady subsonic plane-parallel gas flow in acceleration potential space, obtain the Fredholm equation for determining the distribution function $\Gamma(\theta)$, and find the asymptotic solution.

It is well-known that the singular integral equation for the problem of the oscillations of a profile in an unsteady subsonic gas flow can be obtained for acceleration potential space in the form

$$\frac{1}{2\pi}\int_{-1}^{+1}\gamma(\xi)K(M,x-\xi)d\xi=F(x)\qquad x\in[-1,+1], \tag{1}$$

where the kernel $K(M,x-\xi)$ depends on the Mach number M and the Strouhal number P, while function $F(x)$ is determined by the form of the profile oscillations. We know that in the acceleration potential space kernel $K(M,x-\xi)$ has the form:

$$K(M,x-\xi)=\lim_{y\to 0}\frac{(x-\xi)^2-y^2}{[(x-\xi)^2+y^2]}+K_1(M,x-\xi), \tag{2}$$

where

$$K_1(M,x-\xi)\in C[-1,+1].$$

Then, using expression (2), we can give equation (1) the form:

$$\frac{1}{2\pi}\lim_{y\to 0}\int_{-1}^{+1}\frac{\gamma(\xi)[(x-\xi)^2-y^2]d\xi}{[(x-\xi)^2+y^2]^2}+\frac{1}{2\pi}\int_{-1}^{+1}\gamma(\xi)K_1(M,x-\xi)d\xi=F(x). \tag{3}$$

Transferring the second term on the left side of equation (3) to the right side, we can obtain

$$[illegible] = F(x) - \frac{1}{2\pi}\int_{-1}^{+1}\Gamma(\xi)K_1(M, x-\xi)\,d\xi = F_{эф}. \quad (4)$$

The integral of the differential equation which relates the acceleration potential θ and the velocity potential φ, which satisfy the condition of the absence of perturbations at an infinite distance ahead for a compressible flow, is as follows (5):

$$\varphi = -e^{\frac{ik}{M}x}\int_{-\infty}^{x}\theta e^{-\frac{ik}{M}\tau}\,d\tau. \quad (5)$$

Then we can represent the right side of equation (4) as follows:

$$F_{эф} = \frac{ik}{M}F_{1эф} - F_{1эфx}, \quad (6)$$

where $$F_{1эф} = -e^{\frac{ik}{M}x}\int_{-\infty}^{x}F_{эф}\cdot e^{-\frac{ik}{M}\tau}\,d\tau.$$

Thus, we have arrived at the problem of solving singular integral equation (4) with the right side of (6). Equations of this type are solved in (2).

Finding the solution according to [2], we can write the general expression for distribution function $\gamma(\xi)$:

$$\gamma(\xi) = a_0\sqrt{\frac{1+\xi}{1-\xi}} + 2\lambda\frac{\varkappa}{H}C\sqrt{1-\xi^2} + \frac{2}{\pi}D\int_{-1}^{\xi}\frac{dx}{\sqrt{1-x^2}} + \gamma_1(\xi) - \frac{\lambda\varkappa}{H}\int_{-1}^{\xi}\gamma_1(x)dx, \quad (7)$$

where $$\Gamma_1(\xi) = \frac{2}{\pi} \int_{-1}^{+1} \frac{F_{1\nu\varphi}}{\sqrt{1-x^2}(x-\xi)}\, dx,$$

$C\left(\frac{\varkappa}{M}\right)$ – the Theodorsen function,

$$a_v = 2C\left(\frac{\varkappa}{M}\right)\left[C + \frac{C_1}{2}\right] - C_2; \qquad C_2 = -\frac{2_1}{\frac{\varkappa}{M}\pi} D;$$

$$C + \frac{C_1}{2} = -\int_{-1}^{+1} \sqrt{\frac{1-x}{1+x}}\, F_{1\nu\varphi}\, dx; \qquad D = \frac{i\varkappa}{M} \int_{-1}^{+1} \frac{x\, F_{1\nu\varphi}\, dx}{\sqrt{1-x^2}}$$

$$C = -\frac{1}{\pi} \int_{-1}^{+1} \frac{F_{1\nu\varphi}}{\sqrt{1-x^2}}\, dx.$$

Further, by eliminating that part of $F_{1\nu\varphi}$ which is related to distribution function $\Gamma(\xi)$, we can write expression (7) in the form of the Fredholm integral equation for determining $\Gamma(\xi)$

$$\gamma(\xi) = \gamma_0(\xi) + \int_{-1}^{+1} \gamma(p) K_2(M, \xi, p)\, dp, \qquad (8)$$

where γ_0 is determined by formula (7) for the function of the form of the profile oscillations and kernel $K_2(M, \xi, P)$ - is the following:

$$K_1(M,\xi,P) = \frac{1}{\pi^2}\int_{-1}^{+1}\Big[[\text{illegible}]\frac{x}{M}\Big)\sqrt{\frac{1+\xi}{1-\xi}}\sqrt{\frac{1-x}{1+x}} + \sqrt{\frac{1+\xi}{1-\xi}}\,\frac{x}{\sqrt{1-x^2}} + \frac{ix}{M}\,\frac{\sqrt{1-\xi^2}}{\sqrt{1-x^2}} -$$
$$-\int_{-1}^{1}\frac{dq}{\sqrt{1-q^2}}\,\frac{x}{\sqrt{1-x^2}}\,[\text{illegible}]\,\frac{\sqrt{1-\xi^2}}{\sqrt{1-x^2}(x-\xi)} + \frac{ix}{M}\,\frac{1}{\sqrt{1-x^2}}\int_{-1}^{x}\frac{dq}{x-q}\Big]\cdot e^{\frac{ix}{M}x}\int_{-\infty}^{x} e^{-\frac{ix\tau}{M}} K_1(M,\tau-p)\,d\tau\,dx$$

One of the main advantages of equation (8) for determining the distribution function $\gamma(\xi)$ is that the zero approximation for solving equation (8) is the distribution function γ_0 for unsteady flow, while for all the other equations, this zero approximation serves as the distribution function γ for steady flow.

The form of the kernel $K_1(M, x-\xi)$ is obtained with

consideration of the linearized gas-dynamic movement equation for the acceleration potential, which can assume the following well-known form:

$$\nabla\hat{\theta} + \varkappa^2\hat{\theta} = 0, \tag{9}$$

where $\varkappa = \frac{Mp}{1-M^2}$, $\lambda = \varkappa M$, $\theta = \hat{\theta}\cdot e^{ipt - i\lambda x}$.

In the notations we are using, we have the flow condition:

$$\theta = \frac{1}{2\pi}\int_{-1}^{+1}\gamma(\xi)\frac{\partial}{\partial y}\left[\frac{\pi i}{2}H_0^{(2)}\left(\varkappa\sqrt{(x-\xi)^2+y^2}\right)\right]d\xi.$$

The solution to equation (9) can be written as

$$\theta_y = \frac{i\varkappa}{M}\frac{F_1\, e^{i\lambda x}}{\sqrt{1-M^2}} - \frac{(F_1\cdot e^{i\lambda x})_x}{\sqrt{1-M^2}}$$

Then the kernel of equation (1) will be expressed as

$$K(M, x-\xi) = \frac{\partial^2}{\partial y^2}\left[\frac{\pi i}{2}H_0^{(2)}(z)\right] \quad z = \varkappa\sqrt{(x-\xi)^2+y^2} \tag{10}$$

It is easy to show that the kernel $K(M, x-\xi)$ has structure (2) and that kernel $K_1(M, x-\xi)$, which corresponds to kernel $K(M, x-\xi)$, will be as fcllows:

$$K_1(M, x-\xi) = \frac{\pi i}{2}\frac{\partial^2}{\partial y^2}\left[H_0^{(1)}(\text{[illegible]}) - \frac{(x-\xi)^2-y^2}{((x-\xi)^2+y^2)^2}\right]_{y=0}$$

whereas kernel $K_2(M, x-\xi)$ is obtained in the form:

$$K_1(M,\xi,P) = \frac{1}{2\pi} \int_{-1}^{+1} \Big[\Big(1 - \frac{x}{M} \Big) \sqrt{\ldots}\ [illegible]\ \frac{x}{M} \sqrt{1-\xi^2} \frac{1}{\sqrt{1-x^2}}$$

$$- \frac{i x}{M} \int_{1}^{\xi} \frac{dq}{\sqrt{1-q^2}}\ \frac{x}{\sqrt{1-x^2}}\ [illegible]$$

$$e^{\frac{i x}{M} x} \int_{-\infty}^{x} e^{-\frac{i x}{M} \ldots}\ [illegible]$$

This integral and the expressions for the kernel $K_1(M,\xi,P)$ differ when y = 0 and x = 0; however, it can be replaced by an integral which agrees by introducing the following relationship:

$$\frac{\partial^2}{\partial y^2}\frac{\pi i}{2}H_0^{(2)}(z) = -\frac{\partial^2}{\partial x^2}\frac{\pi i}{2}H_0^{(2)}(z) - \varkappa^2\frac{\pi i}{2}H_0^{(2)}(z),$$

then the kernel $K_2(M,\xi,p)$, after certain transformations with consideration of the value of the integral

$$\int_0^\infty \frac{\pi i}{2}H_0^{(2)} e^{-\frac{i\varkappa\tau}{M}}d\tau = \frac{iM}{\varkappa}\frac{1}{\sqrt{1-M^2}}\ln\frac{1+\sqrt{1-M^2}}{M},$$

is written as follows:

$$K_2(M,\xi,p) = +\frac{1}{\pi^2}\int_{-1}^{+1}\Big\{C\left(\frac{\varkappa}{M}\right)\sqrt{\frac{1+\xi}{1-\xi}}\sqrt{\frac{1-x}{1+x}} + \sqrt{\frac{1+\xi}{1-\xi}}\frac{x}{\sqrt{1-x^2}} + \frac{i\varkappa}{M}\sqrt{1-\xi^2} -$$

$$-\frac{i\varkappa}{M}\int_{-1}^{\xi}\frac{d\eta}{\sqrt{1-\eta^2}}\frac{x}{\sqrt{1-x^2}} - \frac{\sqrt{1-\xi^2}}{\sqrt{1-x^2}(x-\xi)} + \frac{i\varkappa}{M}\frac{1}{\sqrt{1-\xi^2}}\int_{-1}^{\xi}\frac{d\eta}{x-\eta}\Big]\Big\}\Big[\frac{\partial}{\partial x}H_0^{(2)}(z) -$$

$$-\frac{1}{x-p} + \frac{i\varkappa}{M}\frac{\pi i}{2}H_0^{(2)}(z) + \frac{i\varkappa}{M}\sqrt{1-M^2}\ln\frac{1+\sqrt{1-M^2}}{M}e^{\frac{i\varkappa}{M}x} -$$

$$-\frac{\varkappa^2}{M^2}(1-M^2)e^{\frac{i\varkappa}{M}x}\int_0^x\frac{\pi i}{2}H_0^{(2)}(z)e^{-\frac{i\varkappa}{M}\tau}d\tau \cdot e^{\frac{i\varkappa}{M}p}\frac{i\varkappa}{M}\int_\infty^x\frac{e^{-\frac{i\varkappa}{M}\tau}}{\tau-p}d\tau\Big]dx$$

We will consider the approximate solution to equation (8) for small values of parameter $\varkappa$ ($\varkappa \ll 1$). At small values of --, we will write the asymptotic expression for the Hankel function $H_0^{(2)}$, $H_1^{(2)}$.

$$H_0^{(2)}(Z) = \frac{2}{i\pi} \ln \varkappa |x - p| - \frac{2}{i\pi} \ln 2 + \frac{2}{i\pi} C + 1$$

$$\frac{\partial}{\partial x} H_0^{(2)}(Z) = -\varkappa H_1^{(2)}(Z) = -\frac{2i}{\pi (x - p)}$$

Then kernel $K_2(M, \xi, P)$ assumes the form

$$K_2(M, \xi, P) = + \frac{1}{\pi} \int_{-1}^{+1} \Big[[(\frac{x}{M}) \ldots \frac{1+\xi}{1-\xi} \sqrt{\frac{1-x}{1+x}} + \sqrt{\frac{1+\xi}{1-\xi}} \frac{x}{\sqrt{1-x^2}} +$$

$$+ \frac{\mu}{M} \frac{\sqrt{1 - \mu^2}}{\sqrt{1 - \ldots}} \ldots \frac{i\varkappa}{M} \int \ldots \text{[illegible]}$$

$$\frac{1}{\sqrt{\ldots}} \int \ldots \text{[illegible]} \ldots (x - p) \Big\} \ldots$$

$$+ \frac{i\varkappa}{M} \sqrt{1 - M^2} \ldots \text{[illegible]} \ldots e \ldots \frac{\ldots}{r} \ldots \Big] dx$$

At small values of $\frac{x}{M}$ the last integral in square brackets in expression (11) can be represented according to [4]

$$e^{\frac{ix}{M}x}\int_{-\infty}^{x}\frac{e^{-\frac{ix}{M}\tau}\,d\tau}{\tau-p}=C+\frac{\pi i}{2}+\ln\frac{x}{M}+\ln|x-p|. \qquad (12)$$

Substituting (12) in (11) and making the obvious abbreviations, we will obtain the following for small values of parameter $\varkappa$ of kernel $K_2(M,\xi,P)$

$$K_2(M,\xi,P)=+\frac{1}{\pi^2}\int_{-1}^{+1}\Big[C\left(\frac{x}{M}\right)\sqrt{\frac{1+\xi}{1-\xi}}\sqrt{\frac{1-x}{1+x}}+\sqrt{\frac{1+\xi}{1-\xi}}\frac{x}{\sqrt{1-x^2}}+$$

$$+\frac{i\varkappa}{M}\frac{\sqrt{1-\xi^2}}{\sqrt{1-x^2}}-\frac{i\varkappa}{M}\int_1^{\xi}\frac{dq}{\sqrt{1-q^2}}\frac{x}{\sqrt{1-x^2}}-\frac{\sqrt{1-\xi^2}}{\sqrt{1-x^2}(x-\xi)}+ \qquad (13)$$

$$+\frac{i\varkappa}{M}\frac{1}{\sqrt{1-x^2}}\int_1^{\xi}\frac{dq}{x-q}\Big]-\frac{i\varkappa}{M}\left[\ln\frac{M}{2}+\sqrt{1-M^2}\ln\frac{1+\sqrt{1-M^2}}{M}\right]dx$$

Thus, equation (8) can be written as follows for small $\varkappa$:

[illegible]

Integrating expression (14) from -1 to +1, we can immediately obtain the formula for the lift of an oscillating wing in a subsonic compressible flow, retaining the terms whose order of magnitude is not higher than [illegible]

$$[illegible] \quad (15)$$

where $\lambda = \varkappa M$. At $M = 0$, we will obtain the well-known result [1, 2] for an incompressible flow from formula (15):

$$\bar{p} = -2\left[C(p)\int_{-1}^{+1}\sqrt{\frac{1-x}{1+x}}F_1\,dx + ip\int_{-1}^{+1}\sqrt{1-x^2}\,F_1\,dx\right].$$

The comparison of the well-known numerical results with those obtained by formula (15) indicates good agreement.

Bibliography

1. Некрасов А.И. Собрание сочинений. Т.2. Изд-во АН СССР. М., 1962.

2. Панченков А.Н. Плоская задача о колебании крыла в несжимаемой жидкости. Сб. "Судостроение и морские сооружения". Вып.7, 1967.

3. Хаскинд М.Д. Теория крыла, колеблющегося в дозвуковом потоке. Труды ЦАГИ, № 646, 1947.

4. Кочин Н.Е. Собрание сочинений. Т.2 Изд-во АН СССР. М., 1949.

5. Ван-де-Вурен А.И. Теория нестационарного движения крыла. Проблемы механики. Вып. I. Сборник статей под редакцией Х.Драйдена и Т.Кармана. Изд-во ИЛ. М., [illegible].

6. Фын Я.Ц. Введение в теорию аэроупругости. ГИФМЛ. М., 1959.

7. Бисплингхофф Р.Л., Эшли Х., Халфмэн Р. Аэроупругость. Изд-во ИЛ, М., [illegible].

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